Psicothema

Psicothema 2020, Vol. 32, No. 4, 607-614
doi: 10.7334/psicothema2020.179

ISSN 0214 - 9915 CODEN PSOTEG

www.psicothema.com

Bi-factor Exploratory Structural Equation Modeling Done Right: Using the SLiDapp Application

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Abstract

Background: Due to its flexibility and statistical properties, bi-factor Exploratory Structural Equation Modeling (bi-factor ESEM) has become an often-recommended tool in psychometrics. Unfortunately, most recent methods for approximating these structures, such as the SLiD algorithm, are not available in the leading software for performing ESEM (i.e., Mplus). To resolve this issue, we present a novel, user-friendly Shiny application for integrating the SLiD algorithm in bi-factor ESEM estimation in Mplus. Thus, a two-stage framework for conducting SLiD-based bi-factor ESEM in Mplus was developed. **Method:** This approach was presented in a step-by-step guide for applied researchers, showing the utility of the developed SLiDApp application. Using data from the Open-Source Psychometrics Project (N = 2495), we conducted a bi-factor ESEM exploration of the Generic Conspiracist Beliefs Scale. We studied whether bi-factor modelling was appropriate and if both general and group factors were related to each personality trait. **Results:** The application of the SLiD algorithm provided unique information regarding this factor structure and its ESEM structural parameters. **Conclusions:** The results illustrated the usefulness and validity of SLiD-based bi-factor ESEM, and how the proposed Shiny app could make it eaiser for applied researchers to use these methods.

Keywords: Bi-factor, exploratory structural equation modelling, factor analysis, rotation, Schmid-Leiman.

Resumen

Aplicando Bi-factor ESEM: Uso de la Aplicación SLiDapp. **Antecedentes:** los modelos bi-factoriales de ecuaciones estructurales exploratorias (bi-factor ESEM) se han convertido en una herramienta clave en psicometría. Desafortunadamente, las últimas alternativas para su estimación no se encuentran disponibles en el software principal usado para su aproximación (i.e., Mplus). Para solucionar este problema se presenta una aplicación Shiny (SLiDApp) que permite integrar los resultados del algoritmo SLiD en un modelo bi-factor ESEM estimado en Mplus. Para ello, se diseñó una estrategia de dos pasos para aproximar estos modelos. **Método:** este enfoque se ilustró a través de una guía paso por paso de cómo usar la aplicación diseñada y el análisis de un modelo bi-factor ESEM basado en SLiD de la Escala de Creencias Conspirativas Genéricas usando datos del Open-Source Psychometrics Project (N = 2495). Se analizó la relación de los factores generales y de grupo con los cinco factores de personalidad. **Resultados:** los resultados mostraron cómo el algoritmo SLiD proveía de información única acerca de la estructura factorial y los parámetros estructurales. **Conclusiones:** este estudio demostró la utilidad tanto de los modelos bi-factoriales ESEM basados en SLiD cómo de la app propuesta. Se espera así que esta aplicación facilite el uso de dichos métodos por parte de investigadores aplicados.

Palabras clave: bi-factor, modelos de ecuaciones estructurales exploratorias, análisis factorial, rotación, Schmid-Leiman.

The bi-factor model plays today a crucial role in the advancement of psychological theory (Reise et al., 2018) with major applications in personality, intelligence or well-being research (García-Garzón et al., 2019a; Primi et al., 2013, Ruggeri et al., 2020). Furthermore, bi-factor models are routinely explored in test validation and development (e.g., Echeverría et al., 2018). Bi-factor modelling is often applied to the study of multifaceted constructs, either to understand the role of general and specific sources of variances, the reliability of global and subscale composite scores or the extent that general and group factors are related with external criteria (Reise et al., 2018). As such, bi-factor models have become a widespread tool in psychometrics (García-Garzón et al., 2020; Giordano & Waller, 2019; Lorenzo-Seva & Ferrando, 2018).

Received: May 20, 2020 • Accepted: August 5, 2020
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Bi-factor models represent a convenient set of factor models that allow the simultaneous estimation of a general factor (common to all items) alongside several group factors (underlying specific sets of items; Reise et al., 2018). As such, bi-factor models have been recently introduced in the context of Exploratory Structural Equation Modeling (i.e., ESEM; Gomes et al., 2017). ESEM has recently gained popularity as it has been shown to improve parameter estimation when compared with traditional structural equation modelling (Guo et al., 2019; Marsh et al., 2019).

The principal ESEM feature is the introduction of Exploratory Factor Analysis (i.e., EFA) measurement models within a SEM model while retaining global and local fit inspection (Longo et al., 2018), measurement invariance testing (e.g., Lucas-Molina et al., 2017), and the ability to include residual correlations in the measurement model (Nieto et al., 2017; Asparouhov & Muthén, 2009; Garrido et al., 2018). ESEM differs from SEM in that the latter follows a confirmatory approach, where researchers often impose simple structure measurement models (i.e., fixing cross-loadings to zero; Asparouhov & Muthén, 2009). Unfortunately, as long as those restrictions are inconsistent with the data (which is often the case), this approach has been associated with suboptimal parameter estimation. Thus, the use of ESEM has been recommended as a more realistic alternative (Guo et al., 2019; Marsh, 2019).

Accordingly, a decisive step in ESEM is, as in EFA-based methods, the choice of an appropriate rotation method. Such a decision might be of more relevance in this context, as any estimation bias present in the measurement model propagates to other parameters in the model (Guo et al., 2019; Reise et al., 2018).

With regards to bi-factor modelling, several rotation alternatives are currently available (Abad et al., 2017; Asparouhov & Muthén, 2009; García-Garzón et al., 2019b; Giordano & Waller, 2019; Lorenzo-Seva & Ferrando, 2018). In this sense, this article is designed to introduce the use of one of the current state-of-the-art bi-factor rotation methods within ESEM: the Empirical Iterative Target Rotation based on a Schmid-Leiman solution (García-Garzón et al., 2019b). As this method is only available in R software and ESEM is primarily conducted using M*plus* (Muthén & Muthén, 2017), a novel friendly-user Shiny application called SLiDApp was developed to integrate both softwares (https://slidapp.shinyapps.io/SLiD_app/). Its utility is illustrated using a step-by-step guide and an empirical bi-factor ESEM study of the Generic Conspiracist Belief Scale (GCBS; Brotherton et al., 2013) and its relationship with personality traits. For clarity purposes, the term "bi-factor ESEM" will refer to a full-structural SEM including a bi-factor ESEM measurement model, whereas a "bi-factor ESEM measurement model refers exclusively to the ESEM measurement model in bi-factor form.

The SLiD Algorithm

As interest in bi-factor exploratory factor analysis (i.e., bi-factor EFA) has dramatically grown over the last decade, many articles have been concerned with studying their application within ESEM (Asparouhov & Muthén, 2009). The principal software to conduct ESEM is M*plus* (Muthén & Muthén, 2017), which offers three approaches towards estimating bi-factor EFA models in this context: bi-quartimin, bi-geomin (Jennrich & Bentler, 2011, 2012) and the non-iterative target rotation (Reise et al., 2011). Unfortunately, it is well known in the bi-factor EFA literature that these approaches present stringent limitations and fail to provide accurate parameter estimation under most realistic conditions (Abad et al., 2017; García-Garzón et al., 2020; Giordano & Waller, 2019). Accordingly, several alternatives have recently appeared in the literature: the Direct Schmid-Leiman and the Direct Bi-factor (Giordano & Waller, 2019), the Pure Exploratory Bi-factor Analysis (PEBI; Lorenzo-Seva & Ferrando, 2018) and the Empirical Iterative Target Rotation based on a Schmid-Leiman Solution (i.e., SLiD; García-Garzón et al., 2019b).

Amongst those, the SLiD algorithm presents a unique combination of features (García-Garzón et al., 2019b). The SLiD algorithm has been shown to result in both, improved parameter estimation when compared with alternative algorithms (García-Garzón et al., 2019b) and unbiased estimation of general factor reliability under many circumstances (García-Garzón et al., 2020). The SLiD algorithm approximates a simple exploratory bi-factor model in five steps: (a) First, an initial Schmid-Leiman model is estimated, which is known to represent a biased estimation of the bi-factor model of interest (Reise et al., 2011); (b) Second, the initial Schmid-Leiman solution is used to define a partially specified target matrix using an empirical, factor-specific cut-off point based on loadings' differences (García-Garzón et al., 2019b); (c) A first, tentative exploratory bi-factor solution is computed employing a target rotation using the empirically defined target matrix; (d) The estimated bi-factor solution is subsequently refined through repeating steps b and c until convergence (i.e., the target rotation becomes stationary within iterations); (e) Finally, the refined structure is modified so to approximate the identification conditions defined in Asparouhov and Muthén (2009).

An additional benefit of the SLiD algorithm is that it is freely available in open-source software such as R, which facilitates its integration into alternative platforms and applications. Unfortunately, as said before, the SLiD algorithm is not available in M*plus* (Muthén & Muthén, 2017), which is the preferred software to conduct ESEM. Thus, as of today, practitioners wishing to apply a bi-factor ESEM face an uncomfortable situation: (a) to conduct this analysis using a detrimental rotation method such as bi-geomin (the default option in M*plus*), which would lead to biased, incorrect results; (b) to pre-estimate their measurement models using R using the SLiD algorithm and to translate the rotated factor solutions as fixed parameters in a traditional structural equation model; (c) a two-step framework for computing state-of-the-art bi-factor ESEM, where a refined target bi-factor rotation matrix is estimated in R using the SLiD algorithm and is subsequently used in *Mplus* to estimate the ESEM structural model (as in García-Garzón et al., 2019a). Unfortunately, researchers interested in this latter option would be required to be familiarized with both R and *Mplus* softwares. Thus, to bridge the gap between both software, and provide users with an easy pathway to apply this two-step framework to perform SLiD-based ESEM, a novel Shiny app was developed.

SLiDApp: Implementing Modern Bi-factor EFA in ESEM

As previously acknowledged in this journal, methodological innovations such as the SLiD algorithm are only useful to the extent that they are implemented in software available to the general public (Calderón-Garrido et al., 2019). To this end, in recent years, Shiny-based web applications are gaining popularity (e.g., see the application of Nieto et al. (2019) in https://appdim.shinyapps.io/app_dimensionality/). Shiny is an R package that allows developing interactive web tools (Chan et al., 2019). This article introduces *SLiDapp* (*https://slidapp.shinyapps.io/SLid_app/*), a user-friendly Shiny application that provides the refined bi-factor target resulting from the SLiD algorithm in a format ready to be introduced in M*plus* and applied within an ESEM context (Figure 1).

The different steps to use the app and its features are illustrated in Figure 1. These steps are further shown in the "Instructions" panel within the application. The first step is to select a file in TXT, DAT or CSV format, including variables to be analyzed. The next steps are concerned with file characteristics, such as whether variables names are included in the header (step 2) or the separator character applied (step 3). If the dataset contains missing values, the user must specify how they are coded in the input box shown in step 4 (multiple missing values are accepted). Afterwards, step 5 consists of loading the dataset to the SLiDapp using the "Load Data" button. The user can preview the loaded data using the Data Preview box (step 6) or by clicking in "Display data" (step 7).

To start the analysis, the researcher must specify the number of group factors to be extracted (step 8). In this case, a SLiD solution requires at least two group factors to be estimated. Moreover, the maximum number of iterations for the SLiD algorithm can be changed in step 9, so to avoid convergence issues. After deciding on the model dimensionality and number of iterations, the "Run SLiD" option will be now clickable to run the SLiD algorithm (step 10). A progress bar will be shown while SLiD finishes the computation of the target matrix (step 11). Finally, the estimated solution will be printed on the app interface and ready to be copied (step 12) and/or inserted in a M*plus* input file. Interested users can save the estimated target matrix in their computers by indicating a

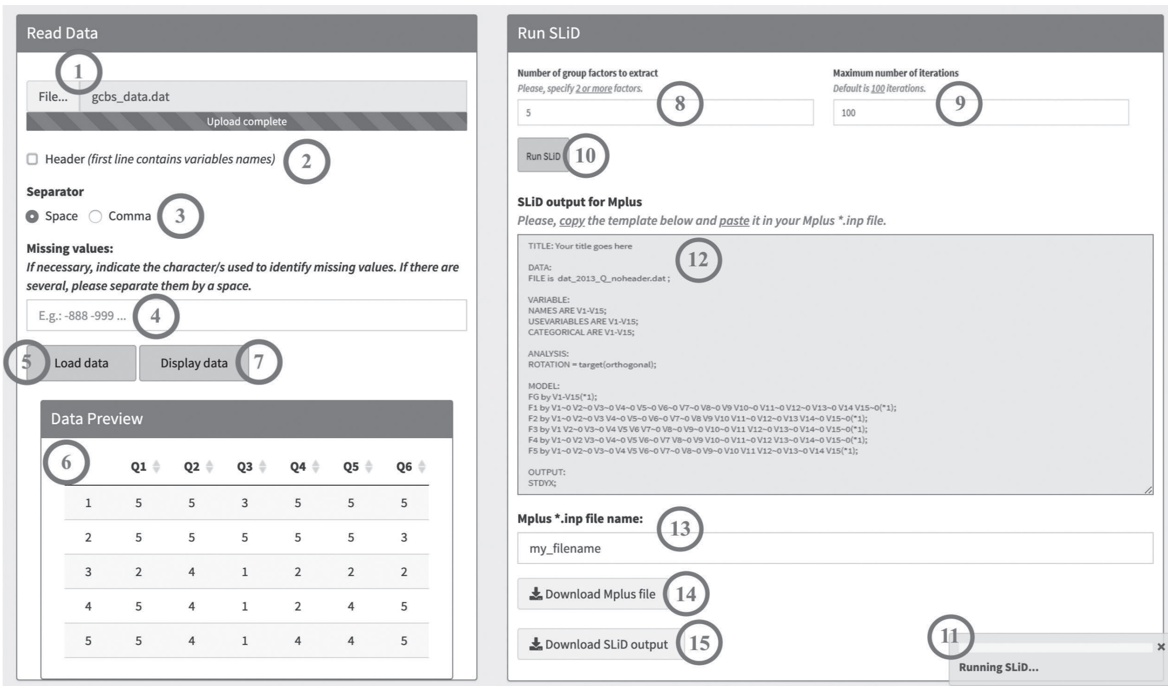

Figure 1. The interface of the Shiny application for computing a SLiD-based target matrix. The different steps for using the app are highlighted and circled in red.

```
TITLE: Your title goes here

DATA:
FILE is  gcbs_data.dat ;

VARIABLE:
NAMES ARE V1-V15;
USEVARIABLES ARE V1-V15;
CATEGORICAL ARE V1-V15;

ANALYSIS:
ROTATION = target(orthogonal);

MODEL:
FG by V1-V15(*1);
F1 by V1~0 V2~0 V3~0 V4 V5 V6~0 V7~0 V8~0 V9~0 V10 V11 V12~0 V13~0 V14 V15(*1);
F2 by V1 V2~0 V3~0 V4 V5 V6 V7~0 V8~0 V9~0 V10~0 V11 V12~0 V13~0 V14~0 V15~0(*1);
F3 by V1~0 V2 V3~0 V4~0 V5 V6~0 V7 V8~0 V9 V10~0 V11~0 V12 V13~0 V14~0 V15~0(*1);
F4 by V1~0 V2~0 V3 V4~0 V5~0 V6~0 V7~0 V8 V9 V10 V11~0 V12~0 V13 V14~0 V15~0(*1);
F5 by V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9 V10~0 V11~0 V12~0 V13~0 V14 V15~0(*1);

OUTPUT:
STDYX;
```

Figure 2. *Mplus* input syntax for conducting bi-factor exploratory factor analysis with a SLiD-based target rotation as produced by the SLIDapp.

name for the resulting file (step 13) and clicking on the "Download *Mplus* file" button (step 14). Furthermore, users can download the SLiD output in CSV format if interested (step 15).

Furthermore, it controls that each line does not exceed 90 characters (a M*plus* restriction). Thus, users only need to add the appropriate code regarding the structural part of the estimated model and to adapt the code of the remaining sections (i.e., DATA, VARIABLE and OUTPUT) in the input and output file (see Supplementary data for a reproducible example using the GBCS data). Whether users are interested only on estimating the SLiD-based exploratory bi-factor model, they can directly run the *Mplus* syntax as provided by the SLiDapp (see Figure 2).

The SLiD algorithm is run automatically using either polychoric or Pearson's correlation based on the number of categories detected in the variables (considered as ordinal when it is either an ordered variable or presents at most, 7 unique integers; Epskamp, 2012), applying the unweighted least-squares extraction method and the oblimin rotation when estimating the initial Schmid-Leiman solution. Missing data is handled using pairwise deletion (i.e., the default option in the fa function from the psych package; Revelle, 2019). The utility of this Shiny app is illustrated below by conducting a SLiD based bi-factor ESEM to the Generic Conspiracist Belief Scale (GCBS; Brotherton et al., 2013) and its relationship with personality traits. This example would investigate both, whether a bi-factor model holds for the GCBS and the relationship between the general and group factors and personality traits derived from the big five models (Goreis & Voracek, 2019).

The GCBS (Brotherton et al., 2013) represents the primary assessment tool in research areas such as inquiring beliefs in fake news, beliefs in conspiracy theories and new forms of information consumption. Accordingly, it has more than 33 research applications in the last five years (Goreis & Voracek, 2019; Hollander, 2018). In this area, there is an increasing controversy surrounding whether conspiracy beliefs are correlated with individual aspects such as personality traits (for a detailed review, see Goreis & Voracek, 2019). While some authors have suggested that higher tendency to believe in conspiracies theories are linked with lower agreeableness and emotional stability and higher openness to experience (Brotherton et al., 2013; Goreis & Voracek, 2019), others argued that such effects were equivocal, to say at least (Goreis & Voracek, 2019).

There are many reasons to believe that the literature surrounding GCBS could benefit from an exploration of bi-factor modelling. Firstly, even though the GCBS was developed as a multidimensional 15-item tool assessing five different conspiracy believes domains (Brotherton et al., 2013), it has been primarily applied as a unidimensional scale assessing a general, conspiracist ideation factor (Hollander, 2018; Swami et al., 2017). Despite the theoretical support for the idea of a general conspiracy ideation factor (Goertzel, 2013; Swami et al., 2017; Wood et al., 2012), evidence showed that unidimensional (or even two-dimensional) GCBS models presented substantive fit issues (Brotherton et al., 2013; Swami et al., 2017). Thus, the latent GCBS structure is still a matter of debate in the literature (Swami et al., 2017). In this sense, a bi-factor model could help to understand the extent that GCBS represents an essentially unidimensional tool and whether the group scales reflect any relevant information additional to this general factor (Reise et al., 2018; Rodríguez et al., 2016).

Method

Participants

Using data from the Open-Source Psychometric Project (www.openpsychometrics.org), responses of 2495 individuals who responded online to the GCBS, the ten-item personality inventory (i.e., TIPI; Gosling et al., 2003) and several demographic items were analyzed. The sample was gender-balanced (females represented 49.0% of the sample), consisted of young aged (M = 27.63, SD = 13.36), higher-educated (36.9% completed university-level studies), English-native speakers (75.2% of participants). Thirteen respondents were removed from the sample due to having response times over 30 minutes response times over two minutes per item. No missing data was observed.

Instruments

The GCBS is a short, 15-item scale that assesses five generic conspiracy domains: government malfeasance, extraterrestrial cover-up, malevolent global conspiracies, personal well-being, and control of information. All items are measured on a five-point Likert Scale to evaluate the veracity of given sentences (i.e., "Evidence of alien contact is being concealed from the public") ranging from 1 ("Definitely not true") to 5 ("Definitely true"). The complete item descriptions are offered in the original manuscript (Table A1; Brotherton et al., 2013).

The TIPI is a brief personality measure which assesses the Big Five personality model (including extraversion, openness to experience, agreeableness, conscientiousness, and emotional stability traits), asking individuals to rate themselves with regards to five positive and five negative adjectives applying a Likert scale ranging from 1 ("Disagree strongly") to 7 ("Agree strongly"). After reversing responses to negative items, each personality trait is measured as the average of the two corresponding items.

Data Analysis

A complete factor-analysis study was conducted to illustrate all the necessary steps appropriate for conducting bi-factor ESEM. Firstly, GCBS dimensionality was estimated employing parallel analysis (Garrido et al., 2013). Afterwards, unidimensional, confirmatory and exploratory versions of the five correlated factors and confirmatory and exploratory bi-factor measurement models with five group factors were analyzed. Secondly, several bi-factor rotation methods were tested, namely the bi-geomin, bi-quartimin, a theory-driven partially specified target rotation, and a SLiD-based target rotation. The quality and reliability of each solution were assessed through omega hierarchical (i.e., ω_H), the expected common variance (i.e., ECV), and the replicability index (i.e., H-index) following Rodríguez et al., (2016) guidelines. Lastly, the bi-factor ESEM model using the SLiD-based target rotation was conducted to estimate the relationship between the different GCBS factors and TIPI personality traits. In these analyses, the SLiD-based target was estimated using the Shiny app. All subsequent analyses were performed using the weighted least-squares with mean and variance correction (WLSMV) in M*plus* 7 (Muthén & Muthén, 2007). Parallel analysis and bi-factor indices were computed in R 3.6.2 (R Core Team, 2019) using the psych package 1.9.12. (Revelle, 2019). Due to data characteristics (i.e., few items per factor and expected high inter-factor correlations) analysis was conducted over the reduced polychoric correlation matrix using the mean eigenvalue rule to decide the number of factors to be retained (Golino et al., 2020).

Results

First Step: GCBS Exploratory Factor Analysis

Even though parallel analysis indicated that five factors should be retained (empirical eigenvalues were 8.30, .83, .40, .14, .08 and .02, and averaged resampled eigenvalues were .36, .12, .11, .08, .06 and .05), the relative size of the first eigenvalue indicates that a dominant dimension might be present. Thus, this dimensionality assessment suggested a combination of a strong single factor altogether with additional minor factors consistent with the hypothesis of a bi-factor model being appropriate for GCBS.

Five different GCBS measurement models were compared in terms of data fit (Table 1), from which an EFA model with five factors fitted the data the best from the set of first-order models. RMSEA was observed to notably differ between CFA and EFA models, which could be attributed to wrongly fixing to zero some cross-loadings, leading to high inter-factor correlation (for CFA, the average inter-factor correlation was of .802; for EFA with oblimin rotation, the average inter-factor correlation was .503; see Supplementary Data). Under these circumstances, the latter was favoured. An exploratory version of this model showed an adequate fit to the data (CFI > .99; TLI > .99; RMSEA < .05), but presented a complex factor pattern (i.e., with 7 out of 15 items presenting cross-loadings larger than .20 in absolute value). The high inter-factor correlations, with 5 out of 10 inter-factor correlations having values over .60, indicate substantive overlap over the five correlated factors. Thus, it was decided to explore the fit of bi-factor models combining a general factor plus five additional group factors.

Table 1
Model fit indices for confirmatory and exploratory models estimated

	χ^2	df	*p*	CFI	TLI	RMSEA
Unidimensional	6760.73	90	.000	.922	.909	.172 (.169 -.176)
CFA 5 factors correlated	1018.30	80	.000	.989	.986	.069 (.065 - .072)
EFA 5 factors correlated	192.56	40	.000	.998	.995	.039 (.034 - .045)
CFA bi-factor	1158.64	75	.000	.987	.982	.076 (.072 - .080)
EFA bi-factor	61.67	30	.001	1.000	.999	.021 (.013 - .028)
ESEM	*115.85*	*75*	*.002*	*1.000*	*.999*	*.015 (.009 - .020)*

Note: χ^2 = Chi-square statistic; df = degrees of freedom; *p*= *p*-value associated with χ^2 test of fit; CFI = Comparative fit index; TLI= Tucker-Lewis index; RMSEA = Root Mean Square Error of Approximation (with 90% confidence interval in parenthesis); CFA = Confirmatory Factor Analysis; EFA = Exploratory Factor Analysis; ESEM = Exploratory Structural Equation Model. ESEM model presented in italics

Second Step: Bi-factor EFA Rotation Algorithms Comparison

Several bi-factor EFA solutions were explored, namely a bi-quartimin, bi-geomin, a model rotated using partial target rotation towards the theoretical structure, and a SLiD-based rotation solution (Table 2). Substantive differences across models were found with regards to group factor loadings pattern: From M*plus* rotation criteria, bi-geomin and the theoretical target produced similar solutions, where items 5, 9 and 14 only presented substantive loadings (larger than |.20|) in the general factor and left the personal well-being factor defined by a single item (Item 4). The only solution properly recovering the personal well-being factor was SLiD with items 9 and 14 substantially loading on it. As SLiD was the structure with a stronger resemblance to the theoretical expectation and given its superior performance in previous simulation studies (García-Garzón et al., 2019b; García-Garzón et al., 2020), it was subsequently retained as the model to be used in ESEM analyses. Noteworthy, all models supported the presence of a reliable general factor (ω_H, ECV and H-index: bi-

Table 2
The Generic Conspiracist Beliefs Scale factor loadings estimate using bi-quartimin, bi-geomin, a theory-based target and SLiD target

	Bi-quartimin						Bi-geomin						Theoretical target						SLiD					
	GCI	GM	MG	ET	PW	CI	GCI	GM	MG	ET	PW	CI	GCI	GM	MG	ET	PW	CI	GCI	GM	MG	ET	PW	CI
I1	**.78**	**.34**	-.03	-.07	-.03	.11	**.76**	**.39**	.00	-.04	-.01	.09	**.74**	**.43**	.03	-.03	.01	.09	**.79**	**.34**	-.02	-.05	-.07	.07
I6	**.80**	**.22**	.03	-.05	.06	-.01	**.79**	**.26**	.05	-.03	.07	.00	**.78**	**.28**	.07	-.02	.07	.00	**.81**	**.20**	.04	-.03	-.01	-.02
I11	**.79**	.18	-.04	-.11	-.08	-.06	**.78**	**.25**	-.03	-.10	-.08	.15	**.76**	**.30**	.00	-.08	-.08	.15	**.78**	**.26**	.01	-.06	.05	.19
I2	**.73**	.09	**.40**	-.03	.00	-.05	**.72**	.10	**.41**	-.02	.01	-.01	**.71**	.11	**.43**	-.01	.02	.00	**.72**	.08	**.42**	.00	-.05	-.01
I7	**.74**	-.04	**.45**	.03	.03	.15	**.75**	-.04	**.45**	.03	.03	-.01	**.74**	-.03	**.45**	.03	.02	.01	**.73**	-.04	**.47**	.06	-.01	.01
I12	**.81**	-.04	**.42**	-.02	-.02	.02	**.81**	-.02	**.42**	-.02	-.03	-.05	**.80**	.00	**.43**	-.02	-.04	-.03	**.78**	.02	**.47**	.03	.06	.01
I3	**.65**	-.01	.03	**.61**	.03	.00	**.65**	-.03	.03	**.61**	.04	-.06	**.65**	-.03	.03	**.60**	.03	-.05	**.63**	-.03	.05	**.63**	.00	-.07
I8	**.63**	.04	-.02	**.67**	.01	.04	**.63**	.02	-.02	**.67**	.02	.04	**.63**	.03	-.01	**.67**	.02	.05	**.62**	.01	-.02	**.68**	-.07	.01
I13	**.70**	-.11	-.02	**.52**	-.03	-.11	**.70**	-.10	-.03	**.51**	-.03	-.03	**.70**	-.08	-.02	**.51**	-.06	-.02	**.65**	-.04	.04	**.57**	.12	.02
I4	**.80**	-.01	.02	.05	**.46**	**.29**	**.81**	.00	.01	.04	**.45**	-.08	**.83**	-.02	-.01	.02	**.40**	-.07	**.87**	**-.21**	-.05	.01	-.01	**-.23**
I9	**.76**	-.18	.05	.12	.00	.17	**.77**	-.14	.03	.11	-.02	-.13	**.77**	-.11	.04	.11	-.08	-.12	**.71**	-.04	.15	.19	**.28**	-.03
I14	**.81**	-.12	-.08	.00	.02	-.04	**.82**	-.07	-.09	-.01	.01	.01	**.82**	-.02	-.08	-.01	-.05	.02	**.78**	-.01	.00	.06	**.25**	.09
I5	**.71**	-.06	-.03	.05	.16	-.03	**.72**	-.04	-.04	.03	.15	.13	**.72**	-.01	-.05	.03	.11	.14	**.73**	-.10	-.04	.05	.06	.10
I10	**.60**	.00	.02	.08	-.01	.04	**.61**	.02	.01	.07	-.01	**.27**	**.60**	.06	.02	.08	-.03	**.28**	**.61**	.00	.02	.09	-.02	**.27**
I15	**.69**	.05	-.03	-.07	-.04	**.48**	**.70**	.09	-.04	-.08	-.03	**.45**	**.69**	.15	-.02	-.08	-.06	**.46**	**.71**	.05	-.04	-.06	-.05	**.45**

Note: GCI: General Conspiracist Ideation. GM: Government Malfeasance; MG: Malevolent Global Conspiracies; ET: Extraterrestrial Cover-up; PW: Personal well-being; CI: Control of Information. SLiD: Empirical Iterative Target Rotation based in a Schmid-Leiman Solution. Factor Loadings with an absolute value over .20 are presented bolded and shadowed in grey quartimin .91/.74/.95; bi-geomin = .92/.76/.95; theory-based target = .91/.75/.95; SLiD = .92./.76/.95).

Third Step: SLiD-based Bi-factor ESEM

After deciding on a bi-factor measurement model, the SLiD-based target matrix was applied within an ESEM framework in *Mplus* via to the Shiny app. In this model, we explored the relationships between the GCBS factors and the Big Five personality traits utilizing ESEM (Figure 3).

The standardized regression parameters for each personality factor and each method (with bi-quartimin, bi-geomin, the theory-based target rotation and the SLiD-based rotation) are presented in Table 3. The model fitted the data excellently (CFA = 1.000; TLI > .99; RMSEA < .02; Table 2). The SLiD-based ESEM model revealed distinct patterns of relationships for each GCBS factor involved, such as the general factor being the only factor significantly (and positively) related with openness ($\beta = .106, p < .001$) and negatively related with both agreeableness ($\beta = -.058, p < .001$) and emotional stability ($\beta = -.076, p < .001$). Nevertheless, the observed relationships presented small predictive power ($.013 < R^2 < .038$). Thus, even though the results from the bi-factor ESEM results aligned with some theoretical predictions, they largely supported previous conclusions suggesting that the relationships between the GCBS factors and personality traits are, at best, weak (Goreis & Voracek, 2019).

Finally, it should be highlighted that whether another bi-factor rotation had been chosen, these results would have been substantially different. Notable disagreements were found among methods, particularly, but not limited, to the personal well-being factor. For example, the relationship between conscientiousness and personal well-being was observed to be significantly positive for bi-quartimin, and non-significant for the remaining methods. Additionally, and the PW for SLiD factor produced a number of estimates in the opposite direction than the remaining methods (e.g., Emotional Stability). In the case of control of information, SLiD was the only method not found to produce a significant relationship between openness and emotional stability and this factor. Thus, these results reflect that the bi-factor rotation determined the nature of the estimated structural parameters.

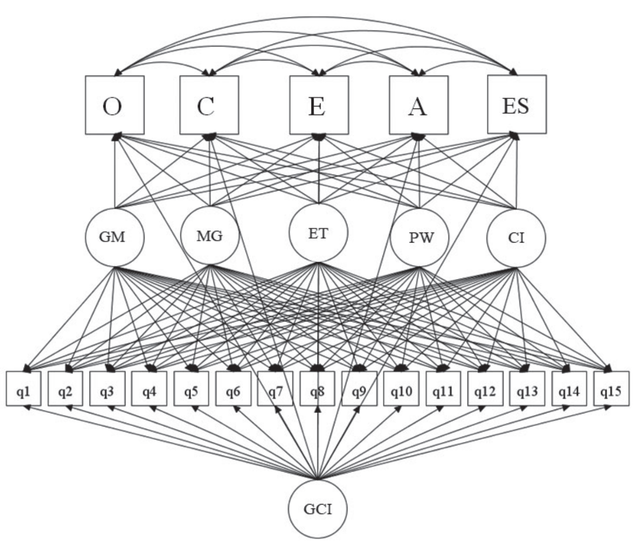

Figure 3. The Bi-factor ESEM model for between the Generic Conspiracist Beliefs Scale scores and personality traits. Items are represented from q1 to 15. GCI: General Conspiracist Ideation. GM: Government Malfeasance; MG: Malevolent Global Conspiracies; ET: Extraterrestrial Cover-up; PW: Personal well-being; CI: Control of Information; O = Openness; C = Conscientiousness; E = Extraversion; A = Agreeableness; ES = Emotional Stability.

Discussion

Bi-factor ESEM models constitute today a crucial tool for latent variable modelling. As such, many researchers who have become

Table 3
Standardized regression coefficients between Big Five personality traits and CGBS scale for different bi-factor rotation methods

	Bi-quartimin	Bi-geomin	Theory Target	SLiD-based
GCI				
Openness	**.095**	**.096**	**.094**	**.106**
Conscientiousness	.016	.020	.030	.020
Extraversion	.042	.047	.049	.036
Agreeableness	**-.056**	-.054	-.049	**-.058**
Emotional Stability	-.054	**-.056**	**-.058**	**-.076**
GM				
Openness	.044	.033	.036	.006
Conscientiousness	-.093	**-.094**	**-.115**	**-.107**
Extraversion	-.071	-.076	-.077	-.059
Agreeableness	-.047	-.057	-.065	-.051
Emotional Stability	-.047	-.039	-.038	.011
MG				
Openness	.024	.024	.024	.002
Conscientiousness	.060	.057	.047	.063
Extraversion	.007	.064	.062	**.078**
Agreeableness	-.008	-.012	-.017	-.009
Emotional Stability	.070	.071	**.076**	**.107**
ET				
Openness	.048	.048	.048	.037
Conscientiousness	-.013	-.020	-.028	-.018
Extraversion	.028	.021	.020	.033
Agreeableness	**.082**	**.078**	**.075**	**.079**
Emotional Stability	**-.100**	**-.099**	**-.096**	**-.078**
PW				
Openness	.008	.014	.021	-.067
Conscientiousness	**.094**	.084	.065	.052
Extraversion	.001	-.005	-.014	.028
Agreeableness	.013	.011	.006	.012
Emotional Stability	-.086	-.055	-.059	.082
CI				
Openness	**.095**	**.095**	**.094**	.072
Conscientiousness	**-.131**	**-.134**	**-.129**	**-.138**
Extraversion	-.018	-.018	-.012	-.033
Agreeableness	-.039	-.036	-.034	-.036
Emotional Stability	**-.109**	**-.108**	**-.104**	-.064

Note: GCI: General Conspiracist Ideation. GM: Government Malfeasance; MG: Malevolent Global Conspiracies; ET: Extraterrestrial Cover-up; PW: Personal well-being; CI: Control of Information. SLiD: Empirical Iterative Target Rotation based in a Schmid-Leiman Solution. Significant regression parameters (at .01 level) presented bolded and shadowed in grey interested in these models in the literature find themselves limited to certain bi-factor rotation methods as these models are primarily estimated using *Mplus*. Despite their popularity, software default methods are not always appropriate, and ultimately impair the advancement of good analytical practices in the context of factor analysis (Izquierdo et al., 2014).

To improve this situation, this research aimed to provide readers with the necessary tools for applying modern bi-factor estimation utilizing the SLiD algorithm within ESEM. We exemplified how to perform this analysis by illustrating the use of a Shiny application and a novel bi-factor ESEM exploration of the relationship between the Generic Conspiracist Beliefs Scale and the Big Five personality traits. Results evidenced the relationships between GCBS factors and personality traits were dependent upon the choice of the bi-factor rotation methods. Moreover, despite supporting latest findings in the GCBS literature (Goreis & Voracek, 2019), it is our understanding that such scale would ultimately benefit from being re-constructed following current directions in the field (Muñiz & Fonseca-Pedrero, 2019).

Lastly, this study was not without limitations. For example, as of today, the Shiny app does not allow users to choose an estimation method or initial rotation method for SLiD. However, future versions of the app will expand these capabilities. Moreover, it should be acknowledged that other relevant bi-factor rotation methods, such as the Pure Exploratory Bi-factor Analyses, were not explored here (as they are only available in specialized software; FACTOR; Ferrando & Lorenzo-Seva, 2017; Lorenzo-Seva & Ferrando, 2018).

With bi-factor ESEM being posed to play a substantial role in future psychological research, we consider of importance to ensure that interested researchers can use state-of-the-art methods regardless of their programming skills. With the same humble spirit that other colleagues have previously expressed in this journal (Calderón Garrido et al., 2019), it could only be hoped that by facilitating the use of these modern methods via a user-friendly, free Shiny app, we have contributed to foster critical thinking and good researcher practices in the context of factor analysis.

Acknowledgements

This work was supported by Grant FPU15/ 03246 from the Ministry of Education, Culture and Sports (Spain), by the Cátedra de Modelos y Aplicaciones Psicométricos (Instituto de Ingeniería del Conocimiento and Autonomous University of Madrid) and by Grant PSI2017-85022-P from Ministerio de Economia y Competitividad (Spain) and Fondo Social Europeo.

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